Shifted Symplectic Semi[n]ar

Fall 2024
Organizers: James Hotchkiss, Lucy Yang

Fridays, 9:00–10:30 am

List of topics, with tentative dates

1. Introduction (Sept. 13)

2. Foundations of DAG

   - The primary reference is §2 and §3 of [Kha24].

3. Shifted symplectic structures. (Oct. 4)

   - §1.2 of [PTVV13]

4. Darboux coordinates (Oct. 11)

   - [BBJ19, BBBBJ15]

5. Purity for 0-shifted symplectic stacks

6. Shifted symplectic structures and DT theory (Oct. 18–Lucy)

   - Review of DT3 via obstruction theories [BF97].
   - Shifted symplectic enhancements and obstruction theories [STV15].
   - Optional: DT4 and (-2)-shifted symplectic structures [OT23].

7. Cohomological DT theory (Nov. 1)

   - Review of the critical CoHA of Kontsevich–Soibelman.
   - Review of vanishing cycles and perverse sheaves
   - Orientations
   - The DT perverse sheaf

   The reference is [Kin24, §1–4].

8. Lagrangian correspondences (Nov. 8)

   - Definition

- Examples
- The local structure theorem from [JS19].
- Joyce's conjecture
- The implication for cohomological Hall algebras

The reference is [Kin24, §5].

9. Cohomological Hall algebras for CY3 categories (Nov. 15, Nov. 22)

- The reference is [KPS24].

10. Other possible topics: (Dec. 6)

- Orientations for moduli of sheaves on CY3's [JU21]
- Shifted symplectic fibrations (e.g., quotients) [Par24]
- Purity for 0-shifted symplectic stacks with good moduli spaces [Dav24], [Hen24]

References

[BBBBJ15] Oren Ben-Bassat, Christopher Brav, Vittoria Bussi, and Dominic Joyce. A 'Darboux theorem' for shifted symplectic structures on derived Artin stacks, with applications. *Geom. Topol.*, 19(3):1287–1359, 2015.

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[BF97] K. Behrend and B. Fantechi. The intrinsic normal cone. *Invent. Math.*, 128(1):45–88, 1997.

[Dav24] Ben Davison. Purity and 2-Calabi–Yau categories. *Invent. Math.*, 238(1):69–173, 2024.

[Hen24] Lucien Hennecart. Cohomological integrality for symmetric quotient stacks. https://arxiv.org/abs/2408.15786, 2024.

[JS19] Dominic Joyce and Pavel Safronov. A Lagrangian neighbourhood theorem for shifted symplectic derived schemes. *Ann. Fac. Sci. Toulouse Math. (6)*, 28(5):831–908, 2019.

[JU21] Dominic Joyce and Markus Upmeier. Orientation data for moduli spaces of coherent sheaves over Calabi-Yau 3-folds. *Adv. Math.*, 381:Paper No. 107627, 47, 2021.

[Kha24] Adeel Khan. An introduction to derived algebraic geometry. https://www.preschema.com/lecture-notes/2023-kias/dagkias.pdf, 2024.

[Kin24]   Tasuki Kinjo. An introduction to cohomological Donaldson–Thomas theory. https://sites.google.com/view/tasuki-kinjo/notes, 2024.

[KPS24]   Tasuki Kinjo, Hyeonjun Park, and Pavel Safronov. Cohomological Hall algebras for 3-Calabi-Yau categories. https://arxiv.org/abs/2406.12838, 2024.

[OT23]   Jeongseok Oh and Richard P. Thomas. Counting sheaves on Calabi-Yau 4-folds, I. *Duke Math. J.*, 172(7):1333–1409, 2023.

[Par24]   Hyeonjun Park. Shifted symplectic pushforwards. https://arxiv.org/abs/2406.19192, 2024.

[PTVV13]   Tony Pantev, Bertrand Toën, Michel Vaquié, and Gabriele Vezzosi. Shifted symplectic structures. *Publ. Math. Inst. Hautes Études Sci.*, 117:271–328, 2013.

[STV15]   Timo Schürg, Bertrand Toën, and Gabriele Vezzosi. Derived algebraic geometry, determinants of perfect complexes, and applications to obstruction theories for maps and complexes. *J. Reine Angew. Math.*, 702:1–40, 2015.